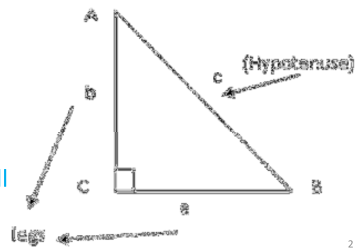
Unit 1

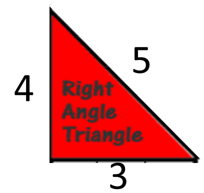
L 1 Pythagorean Theorem

- In a right triangle, the side opposite the 90° angle is called the **hypotenuse** and the remaining two sides are called the **legs**.

Note: Vertices are labeled with CAPITAL LETTERS while, sides are labeled with small letters.

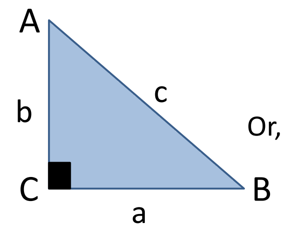


The great Greek Mathematician Pythagoras discovered an interesting relation between the side lengths of the right triangle.



Pythagorean Theorem: If triangle ABC is a right triangle, then $c^2 = a^2 + b^2$

$c = \sqrt{(a^2 + b^2)}$

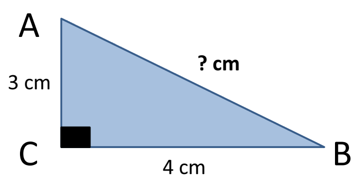


Or, if we are solving for the leg.

$a^2 = c^2 - b^2$

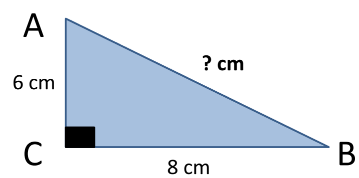
$a = \sqrt{(c^2 - b^2)}$

Ex 1: find the missing side length



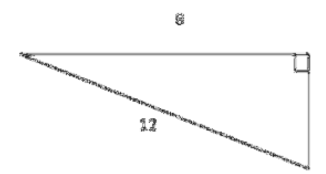
X here is a hypotenuse
So we use **$c^2 = a^2 + b^2$**

Ex 2: find the missing side length



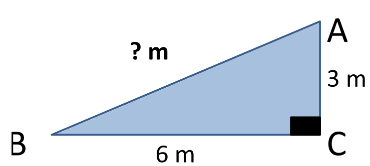
X here is a hypotenuse
So we use **$c^2 = a^2 + b^2$**

Ex 3: find the missing side length



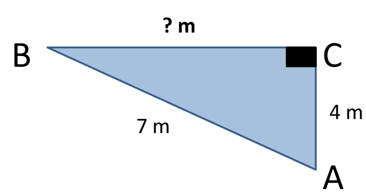
X here is a leg
So we use **$a^2 = c^2 - b^2$**

Ex 4: find the missing side length



X here is a hypotenuse
So we use **$c^2 = a^2 + b^2$**

Ex 5: find the missing side length



X here is a leg
So we use **$a^2 = c^2 - b^2$**

The converse of the Pythagorean Theorem

- The reverse is also true that is if **$c^2 = a^2 + b^2$** then triangle ABC is a right triangle with angle C = 90^o
- Some common Pythagorean triples are:
  {3,4,5} , {5,12,13} , {8,15,17} , {9,40,41}
- and their multiples like
  {6,8,10} , {9,12,15} and {10,24,26} etc.

Practice: 5.2 # 1-13

Unit 1

L2 Number Sets

There are different types of numbers:

- Real Numbers:
  - Natural
  - Integers
  - Rational
  - Irrational
- Complex Numbers (aka Imaginary Numbers)

Definitions:

$\mathbb{N}$: Set of **Natural numbers** : {0,1,2,3,...}

$\mathbb{N}^*$: Set of non zero natural numbers : {1,2,3,...}

$\mathbb{Z}$: Set of **Integers** : {...,-3,-2,-1,0,1,2,3,...}

$\mathbb{Z}^*$: Set of non zero Integers : {...,-3,-2,-1, 1,2,3,...}

$\mathbb{Z}_+$: Set of positive Integers: {0,1,2,3,...) same as $\mathbb{N}$

$\mathbb{Z}_-$: Set of negative Integers: {...,-3,-2,-1,0}

$\mathbb{Q}$: Set of **Rational numbers**

(i.e. numbers that can be written as fractions including terminating decimals (0.5 = $\frac{1}{2}$), and repeating decimals ($0.\overline{5} = \frac{5}{9}$)

Definitions:

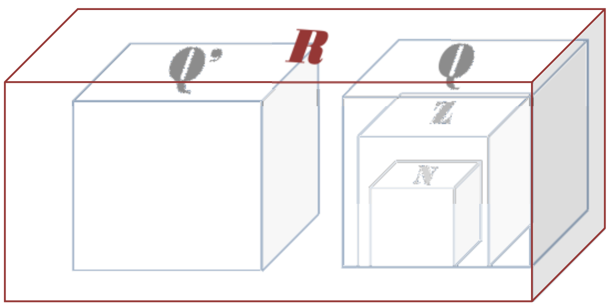
$\mathbb{Q}'$: are Irrational Numbers, these are non-periodic (non-repeating), non-terminating decimals; so we cannot write them as fractions.

(Ex: π , $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{4}$ etc.)

$\mathbb{R}$: is the Set of Real Numbers , that is all Rational and Irrational numbers: **Q** ∪ **Q'**

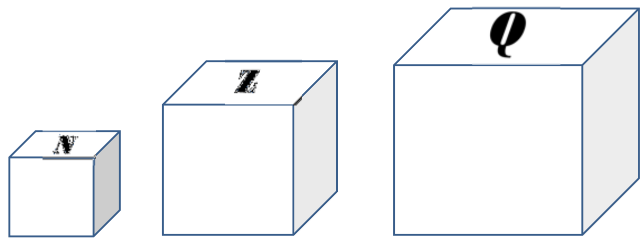
We read this: **Q** Union **Q** prime.

So if we were to put them in nesting boxes (or circles) they would look like this:



Ex 1 : place each number in the correct box.

0 0.3 -3 -7 5 -2/3 100



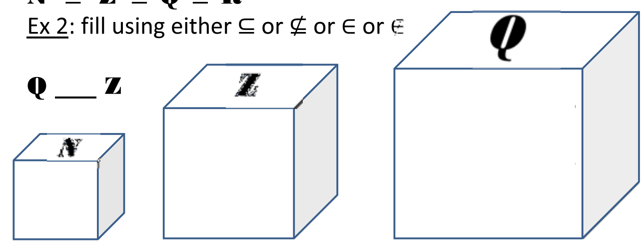
⊆ means subset; ∈ means element of

$\mathbb{N}$ is a subset of $\mathbb{Z}$ is a subset of $\mathbb{Q}$ is a subset of $\mathbb{R}$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

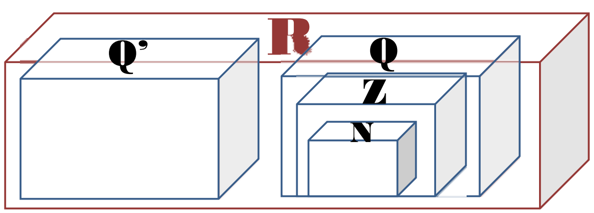
Ex 2: fill using either ⊆ or ⊈ or ∈ or ∉

$\mathbb{Q}$ ___ $\mathbb{Z}$

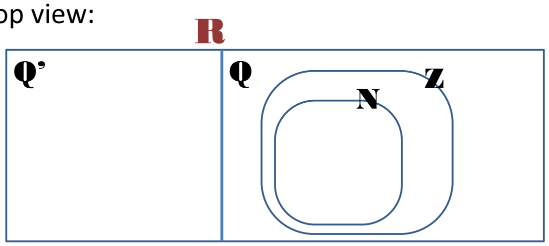


-3 ___ $\mathbb{Z}$

−3 ___ $\mathbb{N}$

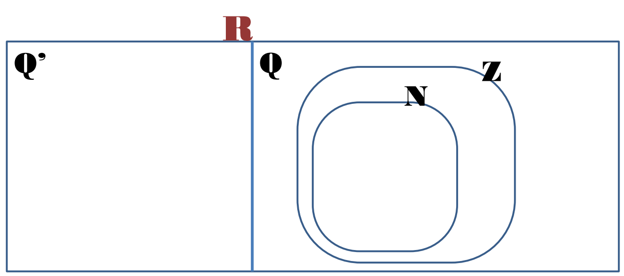


Top view:



Ex 3 : place each number in the correct box.

-1 $-0.\overline{6}$ $-\sqrt{5}$ 11/7 -12 $\sqrt{4}$ 0.5 π 10 $\sqrt{2}$



Practice:
1.1 # 1-4

Unit 1

L3 Writing a Rational number as Fractions or Decimals

Case 1: From Fraction to Decimal

- Use a calculator
- The **period** of a rational number is the infinitely repeating decimal(s) --we put a bar over it.

Ex 1:

a) $\frac{16}{11}$ = the period is

b) $\frac{63}{55}$ = the period is

c) $\frac{1}{2}$ = the period is

Case 2: From Decimal to Fraction

Ex 2: Write the following terminating decimals as reduced fractions.

a) 0.3 =

b) 1.22 =

c) 0.225 =

d) 2.05 =

e) 5.0025 =

f) 3.012 =

Warm up:

1. write as decimals:

a) 1/9= $0.\overline{1}$

b) 2/9= $0.\overline{2}$

c) 3/9= $0.\overline{3}$

d) 4/9= $0.\overline{4}$

e) 5/9= $0.\overline{5}$

f) 6/9= $0.\overline{6}$

g) 7/9= $0.\overline{7}$

h) 8/9= $0.\overline{8}$

2. write as decimals:

a) 1/99 = $0.\overline{01}$

b) 12/99 = $0.\overline{12}$

c) 23/99 = $0.\overline{23}$

d) 45/999 = $0.\overline{045}$

Trick: $\frac{x}{9} = 0.\bar{x}$ and $\frac{xy}{99} = 0.\overline{xy}$

Ex 3: Write the following repeating decimals as reduced fractions.

a) $0.\overline{3}$ =

b) $1.\overline{23}$ =

c) $0.\overline{225}$ =

d) $2.\overline{05}$ =

e) $5.\overline{0025}$ =

f) $3.\overline{012}$ =

Practice:
WS: Practice L3
sets 1-3

If period is not right after decimal point!!

Ex 3: write $0.1\overline{6}$ as a reduced fraction

Method 1:

Reduce and change to improper fraction

$$\boxed{0.1\overline{6}} \xrightarrow{\times 10} 1.\overline{6} = 1\frac{6}{9} = 1\frac{2}{3} = \frac{5}{3} \xrightarrow{\div 10} \frac{5}{30} = \boxed{\frac{1}{6}}$$

We multiply by 10 to get the period alone after the decimal point.

Since that changes the value we have to undo it later by dividing by 10 again.

Dividing by 10 means just add the zero in the denominator.

Method 2: to write $0.1\overline{6}$ as a reduced fraction

Explanation:

1) Make an equation –write the period twice
2) Multiply both sides by a power of 10 to move decimal point to after 1 period
3) and again to before one period.
4) Subtract the 2 equations
5) Solve for x ; and

reduce the fraction if possible

Steps:

1) Let $x = \boxed{0.16\overline{6}}$
2) $100x = 16.\overline{6}$
3) $-\ 10x = 1.\overline{6}$
4) $90x = 15.0$
5) $x = \frac{15}{90}$

$\boxed{= \frac{1}{6}}$

Ex 4: write $1.2\overline{36}$ as a reduced fraction

Method 1:

Method 2:

Practice:
WS: Practice L3
set 4

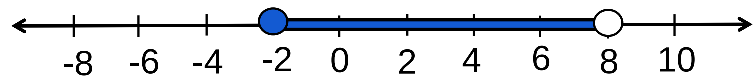
Unit 1

L4 Intervals

Brackets are very important in math and they mean different things. There are 3 types

Round:	P(0,5)	an ordered pair (x=0,y=5)	order is important so, it is not the same as (5,0)
Curly:	S= {0,5}	a set of 2 elements That is 0 & 5 ∈ S	order is not important so, it is the same as {5,0}
Square:	I= [0,5]	an Interval. That is all the real numbers from 0 to 5	same as saying $0 \leq x \leq 5$

Number Line



● Filled: the end number IS in the set

○ Not filled/Empty: the end number IS NOT in the set

▬ Identifies the interval of numbers

][Interval Notation (square brackets)

[-2, 8 [

Lowest # Highest #

[-5, 7]

]5, 9 [

FACING/HUGGING brackets mean the end number is CONTAINED in the set.

BACK FACING brackets means the end number is NOT CONTAINED in the set.

Set builder Notation {Inequalities}

Review: fill in the correct sign so that x is

Less than 5 ____________

Greater than 10 ____________

At Most 22 ____________

At Least 15 ____________

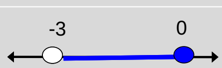
We read the inequality from left to right.

$\{x \in R \mid -2 \leq x < 8\}$

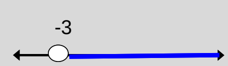
x is a real # Lowest # Highest #

EFF RULE:
Equal sign
Filled circle
Facing bracket

Bounded Intervals

Interval	Set-Builder	Number Line
[0,3]		
	$\{x \in R \mid -1 \leq x < 5\}$	
		
	$\{x \in R \mid a < x < b\}$	

Unbounded Intervals

Interval	Set-Builder	Number Line
]-∞,3]		
	$\{x \in R \mid x \geq 3\}$	
		
	$\{x \in R \mid x < b\}$	

Practice: 3.3 # 1,2

Unit 1

L 5 Exponent Basics

Expanded form : $2\times2\times2\times2\times2\times2\times2 = 128$

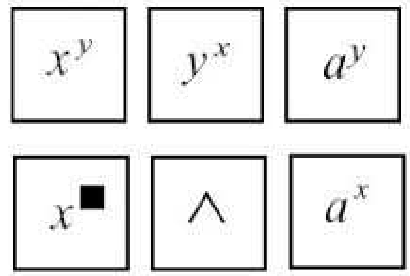
Exponential form:

Exponent

Base → $2^7 = 128$ ← Power

$\therefore \quad a^n = a\times a\times \ldots \times a \quad$ (n times)

Scientific Calculator
Exponent Buttons



Note that:

$a^1 = a$ $\qquad a^0 = 1$

$a^n =$ **if** the 'a' is negative and 'n' is odd

Ex: $(-2)^3 = (-2)(-2)(-2) = -8$

$(-a)^n = (-a)\times(-a)\times \ldots \times(-a) \quad$ (n times)

$-a^n = -a\times a\times \ldots \times a \quad$ (n times)

Ex 1: Expand and evaluate:

a) $7^3 =$

b) $x^4 =$

c) $9^1 =$

d) $5^0 =$

e) $(-8)^4 =$

f) $-8^4 =$

g) $(7)(6)(7)(6) =$

Ex 2: Evaluate:

a) $2 \times 10^3 + 6 \times 10^2 + 7 \times 10 + 3$

b) $\dfrac{2^4 + 5^2}{3^3}$

Practice:
1.2 # 1--9

Unit 1

L6 Laws of Exponents

Question	Expanded Form	Exponent	Law
$5^2 \times 5^4$			**a^m x a^n =**
$2^5 \div 2^2$			**$a^m \div a^n$ =** If m>n
$2^2 \div 2^5$			**$a^m \div a^n$ =** If m<n
$(4^3)^2$			**$(a^m)^n$ =**
$(5\,a)^4$			**$(ab)^n$ =**
$\left(\frac{3}{4}\right)^2$			$\left(\frac{a}{b}\right)^n$ **=**

Why is $a^0 = 1$

$$5^3 \div 5^3$$
$$= \frac{5 \times 5 \times 5}{5 \times 5 \times 5}$$
$$= \frac{125}{125}$$
$$= 1$$

$$5^3 \div 5^3$$
$$= 5^{3-3}$$
$$= 5^0$$

Therefore $5^0 = 1$

Ex 1: Simplify these exponents (DO NOT EVALUATE)

$9^7 \times 9^6 =$ ______

$3^7 \times 3 =$ ______

$4^7 \times 4^{-3} =$ ______

$7^0 \times 7^{12} =$ ______

$2^{22} \times 2^{-20} =$ ______

$(-6)^7 \times (-6)^6 =$ ______

$3^7 \div 3 =$ ______

$9^7 \div 9^6 =$ ______

$4^7 \div 4^{-3} =$ ______

$\frac{7^{12}}{7^9} =$ ______

$2^{22} \div 2^{-20} =$ ______

$\frac{(-6)^5}{(-6)^7} =$ ______

Ex 2: Simplify these exponents

$\frac{3a^2}{9a^4}$ ______

$\frac{15a^5}{25a^2}$ ______

$\frac{3a^4}{6b^2} \times \frac{2a^2}{b^4} \times \frac{b^9}{a^5}$ ______

Ex 3: Simplify these exponents

$(2^7)^2 =$ ______

$(3^3)^2 =$ ______

$(a^4)^3 =$ ______

$(q^n)^m =$ ______

$(5^5)^5 =$ ______

$((-3)^2)^4 =$ ______

Ex 4: Simplify these exponents

$\frac{(x^3)^2}{x^2} =$ ______

______

$\frac{(2^3)(a^2)^3(b^3)^3}{4a^6b^8}$ ______

______

Ex 5: Simplify

$(3a)^6 =$ ______

$(6b)^3 =$ ______

$(2a^3)^2 =$ ______

$(a \times b)^n =$ ______

$\left(\frac{5}{3}\right)^4 =$ ______

$\left(\frac{7}{x}\right)^3 =$ ______

$\left(\frac{b}{8}\right)^2 =$ ______

$\left(\frac{a}{b}\right)^n =$ ______

Ex 6: Express each power as a new power with the given base

$16^2 =$ __________ $= 2^{\_\_}$

$16^2 =$ __________ $= 4^{\_\_}$

$25^3 =$ __________ $= 5^{\_\_}$

$27^3 =$ __________ $= 3^{\_\_}$

Ex : Simplify $4^4 \div 2^2 =$

Ex 7: Fill in with = or ≠

1) $5^2 + 5^3$ $\quad 5^5$
2) $2^4 \times 2^3$ $\quad 2^{12}$
3) $4^2 \times 5^3$ $\quad 20^5$
4) $5^2 + 5^2$ $\quad 10^4$
5) $5^6 - 5^2$ $\quad 5^4$
6) $6^4 / 2^2$ $\quad 3^2$
7) $5^6 / 5^2$ $\quad 5^3$
8) x $\quad x^0$

Ex 8: Simplify

a) $(5x^3)^2 =$ __________

b) $(3a^4b^2)^2\,(-a^5b)^2\,(2a^3b^2)^2$

$=$ ____________________

$=$ ____________________

c) $\left(\dfrac{3x^2}{y^3}\right)^3\left(\dfrac{y^4}{9x^4}\right)^2 =$ ____________________

Ex 9: Extra Practice - Simplify

Evaluate at a = -2 and b = -1

a) $\dfrac{(3x^3y^5)^2}{3x^2y^4}$

$=$ ____________________

b) $\dfrac{(4a^2b^3)^3}{(2a^3b^4)^2}$

____

c) $\dfrac{(5a^6b^4)^3}{(5a^2)^3}$

$=$ ____________________

d) $\dfrac{(6x^2y^3)^3}{(2x^4y^2)^2}$

____

Practice:
Day 1: 1.3 # 1-5
Day 2: 1.4 # 1-4
Day 3: 1.5 # 1-3

Unit 1

L7 Negative Exponents

Warm up: Evaluate using the calculator

2^2 =	2^{-2} =
-2^2 =	-2^{-2} =
$(-2)^2$ =	$(-2)^{-2}$ =
$(2^{-2})^2$ =	$\left(\frac{1}{2}\right)^{-2}$ =

L7 Negative Exponents

$$x^3 = x \cdot x \cdot x$$
$$x^2 = x \cdot x$$
$$x^1 = x$$
$$x^0 = 1$$
$$x^{-1} = 1 \div x = \frac{1}{x^1}$$
$$x^{-2} = 1 \div x \div x = \frac{1}{x \cdot x} = \frac{1}{x^2}$$
$$x^{-3} = 1 \div x \div x \div x = \frac{1}{x \cdot x \cdot x} = \frac{1}{x^3}$$
$$x^{-n} = \frac{1}{x^n}$$

We Divide by x each time

Another way to see what a negative exponent means…

$$4^3 \div 4^5$$
$$= \frac{\cancel{4}\times\cancel{4}\times\cancel{4}}{\cancel{4}\times\cancel{4}\times\cancel{4}\times 4\times 4}$$
$$= \frac{1}{4^2}$$

$$4^3 \div 4^5$$
$$= 4^{3-5}$$
$$= 4^{-2}$$

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

What I call FLIP AND SWITCH
FLIP the base, and SWITCH the exponent to positive

A base raised to a NEGATIVE exponent is equivalent to 1 over the original base with the same exponent but positive .

EX 1: Write with a positive exponent

a) 3^{-2} = ______	d) p^{-3} = ______
b) 2^{-2} = ______	e) $\frac{a^3}{a^7}$ = ______
c) $(10^2)^{-2}$ = ______	f) $\frac{b^5}{b^{10}}$ = ______

More examples

$\left(\frac{3a^{-2}}{b^4}\right)^{-2}$ $\left(\frac{3^3 3^{-2}}{3^{-1}}\right)^{-2}$ 0.01

More examples

$(0.01)^2(0.1)^{-1}$ $(a^2b^{-1})(a^{-2}b^3)$ $\left(\frac{a^4}{2b^{-2}}\right)^{-3}\left(\frac{4b^{-1}}{a^8}\right)^{-2}$

Practice:
1.6 # 1 – 4

Unit 1

L8 Scientific Notation

Warm up: Perform the following operations

1) $14 \div 10 =$

2) $482 \div 1000 =$

3) $662 \div 10^2 =$

4) $127 \times 10 =$

5) $48023 \times 10^4 =$

L8 Scientific Notation

When numbers get really Big or really small it is inconvenient to write out all of the zeros.

For this reason we use *Scientific Notation.*

A positive number in **scientific notation** is in the form:

$a \times 10^n$ where $1 \le a < 10$; and n is an integer.

Ex 1: Express in scientific notation $a \times 10^n$ $1 \le a < 10$

POSITIVE EXPONENT	NEGATIVE EXPONENT
a) 5600	b) 0.00042

Ex 2: Write the following numbers in scientific notation:

1) 360 =

2) 0.4 =

3) 7523 =

4) 45000 =

5) 235 =

6) 1226000 =

7) 0.025 =

8) 0.000045 =

9) 81 =

Examples 3:

42 000 000 000 000 x 72 000 000 000

Examples 4:

125 000 000 000 ÷ 0.000 000 000 25

Examples 5:

42 000 000 000 x 0.000 000 000 21

Examples 6:

42 000 000 000 ÷ 0.000 000 000 126

Practice:
1.8 # 1,2
1.9 # 1-5

Unit 1

L9 Rational Exponents

Warm up:

1) 7^2 =

5) 5^4 =

9) $\sqrt[3]{8}$ =

2) $\sqrt{49}$ =

6) $\sqrt[4]{625}$=

10) $\sqrt[4]{81}$ =

3) 3^3 =

7) $(-3)^3$ =

11) $\sqrt[4]{-16}$ =

4) $\sqrt[3]{27}$ =

8) $\sqrt[3]{-27}$ =

12) $\sqrt[3]{-125}$ =

L9 Rational Exponents

If $b^n = a$ **then** $\sqrt[n]{a} = b$

or $a^{\frac{1}{n}} = \sqrt[n]{a} = b$

Note that $\sqrt[n]{a}$ does not exist if n is even and a < 0

Example: $\sqrt[2]{-64}$ and $\sqrt[4]{-64}$ don't exist

Where as $\sqrt[3]{-64}$ = -4, because
$(-4)^3 = (-4)(-4)(-4) = -64$

Ex 1: Evaluate:

1) $169^{\frac{1}{2}}$ =

2) $(-169)^{\frac{1}{2}}$ =

3) $-169^{\frac{1}{2}}$ =

4) $125^{\frac{1}{3}}$ =

5) $(-125)^{\frac{1}{3}}$ =

Ex 1: Evaluate:

6) $-125^{\frac{1}{3}}$ =

7) $\sqrt[3]{2^3}$ =

8) $144^{-\frac{1}{2}}$ =

9) $\sqrt[3]{-\frac{8}{27}}$ =

Ex 2: Simplify:

a) $\sqrt[3]{\frac{8x^9}{x^3}}$

b) $\left(\frac{9a^{-2}}{b^4}\right)^{-\frac{1}{2}}$

Practice:
1.7 # 1 - 3